H_{\infty} PID controller design for runaway processes with time delay

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(Received 16 April 2001; accepted 16 September 2001)

Abstract

This paper presents an efficient method for designing PID controllers for runaway processes with time delay. The method is developed based on the H_{\infty} control theory in frequency domain. The constraints imposed by the internal stability and asymptotic properties of the closed-loop system are first investigated, a new procedure is then developed for analytically designing the controller, and simple design formulas are obtained. It is shown that the new controller can be designed to meet specified time domain performances. Typical design examples are provided to illustrate the proposed method. © 2002 ISA—The Instrumentation, Systems, and Automation Society.

Keywords: H_{\infty} control; Optimal control; Runaway process; Time delay; PID controller

1. Introduction

Process models are typically expressed as transfer functions falling into three categories [1]: self-regulating processes, integrating processes, and runaway processes. For self-regulating processes, many design methods have been developed. However, there are a few methods about the control of the other two processes, especially, the runaway process with time delay. In a runaway process, there exist right half plane poles which characterize open-loop instability. Due to the combined effects of the right half plane poles and the time delay, the formulation of satisfactory loop compensations is very difficult for runaway processes with time delay. This makes the design task challenging.

Perhaps the control problem of runaway processes with time delay was originally proposed by Luyben and Melcic [2]. In recent decades, several different methods have been developed. In Ref. [3], two design methods for P and PI controllers were proposed based on optimal gain margin and optimal phase margin. These methods were later improved in Ref. [4], which provided an approximate solution for calculating the controller settings. Based on the internal model control, Ref. [5] proposed a method for designing PID controllers. The time delay was neglected and then the controller was designed. Reference [6] also proposed a method. The controller was designed by selecting a suitable closed-loop model. As these methods often provide excessive overshoot, the setpoint filtering method [7,8] was developed in order to get less overshoot.

The PID controller has been widely used in industry since it is simple and robust. This paper attempts to develop a new PID controller design method for runaway processes with time delay. Based on H_{\infty} control theory, an optimal performance index is defined. The constraints imposed by the internal stability and asymptotic tracking of the closed-loop system are investigated. With a low-order rational approximation of the time de-
lay, the PID controller is analytically derived. The stability and performance of the closed-loop system are discussed. It is shown that the new method can provide quantitative time domain performances, such as overshoot and settling time. The effects of the rational approximation and the limitations of the internal model control structure on implementing the new scheme are also discussed.

This paper is organized as follows. Section 2 investigates the condition that guarantees the internal stability and asymptotic tracking of the closed-loop system. In Sec. 3, a new design procedure is developed for deriving the controller. The stability and performance of the closed-loop system are discussed in Sec. 4. The new method is compared with several developed control strategies. Finally, some conclusions are given in Sec. 5.

2. Constraints of the closed-loop system

Consider the internal model control structure shown in Fig. 1, where \( Q(s) \) is the internal model controller, \( G(s) \) is the plant, and \( G_m(s) \) is the model. Let \( C(s) \) represent the controller of a unity feedback loop. The internal model control structure can be equivalent to the unity feedback loop through [9]

\[
C(s) = \frac{Q(s)}{1 - G(s)Q(s)}. \tag{1}
\]

The transfer function of a runaway process can be expressed as

\[
G(s) = \frac{K}{(\tau_1 s - 1)(\tau_2 s + 1)} e^{-\theta s}, \quad \theta \neq \tau_1. \tag{2}
\]

It has been pointed out that the controller \( C(s) \) cannot be analytically designed if the time delay involved in the process is treated strictly [10]. The only possible way is the use of rational approximations [11,12]. When the first-order Taylor series is introduced as an approximation of the time delay, the plant becomes

\[
G(s) = \frac{K(1 - \theta s)}{(\tau_1 s - 1)(\tau_2 s + 1)}. \tag{3}
\]

Under the nominal condition, the sensitivity of the closed-loop system is given by

\[
S(s) = \frac{1}{1 + G(s)C(s)} = 1 - G(s)Q(s) \tag{4}
\]

and the complementary sensitivity (i.e., the closed-loop transfer function) is

\[
T(s) = \frac{G(s)C(s)}{1 + G(s)C(s)} = G(s)Q(s). \tag{5}
\]

To guarantee the internal stability of the closed-loop system, \( Q(s) \) should be stable.

The disturbance is frequently encountered in practical processes. As a basic closed-loop performance specification, the error is often required to vanish asymptotically. In order to meet the specification, \( Q(s) \) should also satisfy the following constraints:

\[
\lim_{s \to 0} S(s) = \lim_{s \to 0} 1 - G(s)Q(s) = 0, \tag{6}
\]

\[
\lim_{s \to 1/\tau} S(s) = \lim_{s \to 1/\tau} 1 - G(s)Q(s) = 0. \tag{7}
\]

3. Design procedure

Define the \( H_\infty \) optimal performance index as \( \min \| W(s)S(s) \|_\infty \), where \( W(s) \) is the weighting function. In process control, \( W(s) \) can be selected as \( 1/s \), which implies that the system input is a unit step signal (see, for example, Ref. [13]).

In the conventional \( H_\infty \) control theory, the design problem is usually converted into a model matching problem by the coprime factorization of the plant, and then the problem is solved by an interpolation method or a loop shaping method [13,14]. Here, we will develop an analytical design method. Recall the well-known maximum modulus theorem; a bounded function cannot attain its maximum value at the interior point. Therefore we get
\[ \|W(s)S(s)\|_\infty = \|W(s)[1 - G(s)Q(s)]\|_\infty \geq |W(1/\theta)|. \] (8)

Minimizing the left side of the inequality yields
\[ W(s)[1 - G(s)Q(s)] = \theta. \] (9)

The optimal \( Q(s) \) is then obtained as follows:
\[ Q_{im}(s) = \frac{(\tau_1 s - 1)(\tau_2 s + 1)}{K}. \] (10)

Obviously, the \( Q_{im}(s) \) is improper and cannot be realized in practice. A filter should be introduced to make it proper. This may follow into two cases.

1. \( \tau_2 = 0 \)
   In this case, the filter should have the pole-zero excess of 1. It can be verified that a first-order filter does not satisfy the constraints of the internal stability and asymptotic tracking. Suppose that the filter has the transfer function
   \[ J(s) = \frac{a_1 s + 1}{(\lambda s + 1)^2}, \] (11)
   where \( \lambda > 0 \) is a constant. We obtain that
   \[ a_1 = \frac{\lambda^2 + 2\lambda \tau_1 + \theta \tau_1}{\tau_1 - \theta}. \]
   It follows that
   \[ Q(s) = Q_{im}(s)J(s) = \frac{(\tau_1 s - 1)(a_1 s + 1)}{K(\lambda s + 1)^2}. \] (12)

The corresponding controller of the unity feedback loop can be written as
\[ C(s) = \frac{b_1}{K} \left( 1 + \frac{1}{a_1 s} \right), \] (13)
where
\[ b_1 = \frac{\lambda^2 + 2\lambda \tau_1 + \theta \tau_1}{(\lambda + \theta)^2}. \]

2. \( \tau_2 \neq 0 \)
   The filter can be chosen as
   \[ J(s) = \frac{a_2 s + 1}{(\lambda s + 1)^2}, \] (14)
   Elementary computations give
   \[ \alpha_2 = \frac{\lambda^3 + 3\lambda \tau_1 + 3\lambda \tau_1^2 + \theta \tau_1^2}{\tau_1(\tau_1 - \theta)}. \]
   This leads to
   \[ Q(s) = Q_{im}(s)J(s) = \frac{(\tau_1 s - 1)(\tau_2 s + 1)(a_2 s + 1)}{K(\lambda s + 1)^3}. \] (15)

The controller of unity feedback loop is as follows:
\[ C(s) = \frac{1}{K} \left( \frac{\tau_2 s + 1}{s} \cdot \frac{\lambda}{\tau_1 s + b_2} \right), \] (16)
where
\[ b_2 = \frac{\lambda^3 + 3\lambda^2 \tau_1 + 3\lambda \tau_1 \theta + \theta^2 \tau_1}{\tau_1(\tau_1 - \theta)}. \]
This is a PID controller. Suppose that the PID controller structure is given by
\[ C(s) = K_C \left( 1 + \frac{1}{T_1 s} + \frac{1}{T_D s} \right) \frac{1}{T_F s + 1}. \] (17)

The parameters can be written as
\[ K_C = \frac{\tau_2 + a_2}{K b_2}, \quad T_1 = \tau_2 + a_2, \quad T_D = \frac{\tau_2 a_2}{\tau_1 + a_2}, \quad T_F = \frac{\lambda^3}{\tau_1 b_2}. \]
where \( \lambda \) can be regarded as an adjustable parameter of the controller. \( \lambda \) relates to the performance and robustness of the closed-loop system directly.

4. System analysis

In the above discussion, the controller is derived by the approximate model. When the controller is used for the control of the original process, the closed-loop transfer function can be expressed as
\[ T(s) = \frac{b_1(a_1 s + 1)e^{-\theta_1}}{s(\tau_1 s - 1)a_1 + b_1(a_1 s + 1)e^{-\theta_1}}. \] (18)
for the first-order process and
For the second-order process, note that the zeros of the second part of the denominator in the two closed-loop transfer functions are

\[ s_1 = -1/a_1, \quad s_2 = -1/a_2, \quad s_3 = -1/t_2. \]

If \( \theta > \tau_1 \), then \( s_1 \) and \( s_2 \) are at right half plane. When \( s \) tends to be \( s_1 \) or \( s_2 \), the closed-loop system tends to be unstable. Therefore the proposed PI and PID controller can only be used for the control of unstable processes with \( \theta < \tau_1 \). Such a condition was previously derived by Ref. [3] with a more complicated method.

For a first-order process, further research shows that when \( \theta/\tau_1 \) is fixed the performance of the closed-loop system is only related to \( \lambda/\theta \). Hence the time domain performance of the closed-loop system can be estimated quantitatively. Define the perturbance peak as the peak of system output when a unit step disturbance is added at the process input, and recovery time be the perturbed system output coming within 5% of its final value. The quantitative performance is shown in Figs. 2–5. As for a second-order process, the system response relates not only to \( \lambda/\theta \) and \( \theta/\tau_1 \), but to \( \theta/\tau_2 \). If \( \theta/\tau_2 \) is fixed, the performance can also be estimated quantitatively.

It is of great importance that the controller performs well even when the dynamic behavior of the real process differs from that described by its model. Suppose that the nominal closed-loop transfer function is \( T(s) \) and the uncertainty profile is \( \Delta(s) \), then the sufficient and necessary condition that guarantees the stability of closed-loop system is [9,13]

\[ \| T(s)\Delta(s) \|_\infty = \| G_m(s)Q(s)\Delta(s) \|_\infty < 1. \] (20)

Thus the performance and robustness of the closed-loop system can be adjusted by \( \lambda \) monotonously.

In conventional design methods, we seldom use rational approximations since they will cause the stability problem [15]. Such a problem does not
exist in the proposed method. Regard the error introduced by the rational approximation as a kind of uncertainty, i.e., \( \Delta(s) \) consists of only the error, then the closed-loop system is stable when the parameter \( \lambda \) is greater than a certain low bound. If there exist other uncertainties, \( \Delta(s) \) will consist of both the error and the uncertainties. The permitted uncertainty in addition to that caused by the rational approximation can be numerically calculated by the above sufficient and necessary condition for a certain process and uncertainty. If \( \theta \tau \leq 0.5 \), we recommend to select \( \lambda = 2 - 5 \theta \) for the proposed PI controller and \( \lambda = 2 - 5(\theta + \tau_2) \) for the proposed PID controller. For \( \theta \tau \) larger than 0.5, \( \lambda \) should be selected larger. This is a rule of thumb. One can also determine \( \lambda \) by observing the closed-loop response for different \( \lambda \).

**Example 1.** For the purpose of comparison, a first-order process from Ref. [2] is used, which has the following transfer function:

\[
G(s) = \frac{e^{-0.5s}}{s-1}.
\]

The controller parameters of the D-O method [3] are \( P = 1.357 \) and \( I = 6.944 \). The controller parameters of the V-C method [4] are \( P = 1.51 \) and \( I = 12.5 \). And the controller settings taken from the R-L method [5] are \( P = 1.667 \) and \( I = 5 \). For the new controller we take \( \lambda = 2 \). The responses of closed-loop system are shown in Fig. 6. As we can see, the D-O method and the R-L method have excessive oscillations, and the V-C method has slow response. The new controller provides fast and steady set-point response and disturbance rejection.

**Example 2.** In order to explain the effects caused by \( \tau_2 \), consider the following two second-order processes:

\[ A: \quad G(s) = \frac{e^{-0.5s}}{(s-1)(0.5s+1)}; \]
\[ B: \quad G(s) = \frac{e^{-s}}{(2s-1)(s+1)}. \]

It might as well let \( \lambda = 2 \). The response is shown in Fig. 7. It is seen that that the system has the same overshoot and perturbation peak, as well as settling time/\( \tau_1 \) and recovery time/\( \tau_1 \).

In a one-degree-of-freedom control system of runaway processes with time delay, the overshoot is usually very large. One efficient way to overcome the problem is to introduce a set-point prefilter [7,8]. If the prefilter is chosen as the inverse of the minimum phase part of the closed-loop transfer function [9], the overshoot will be reduced to zero. If a low-order approximate inverse is used, a small overshoot will be obtained.

5. **Conclusions**

Based on the \( H_\infty \) control theory, a new PID controller is designed for runaway processes with time delay and several results are obtained:

1. The conditions that guarantee the internal stability and asymptotic properties of the closed-loop system are derived.
2. Instead of coprime factorization and the numerical method, a new procedure is developed in frequency domain for the \( H_\infty \) controller design.
3. The settings of the PID controller are analytically derived by Taylor series. It is proved that the
error introduced by the rational approximation will not cause instability in the proposed method.

4. The new scheme relates to the time domain performance closely. The controller can be designed to meet the specification of overshoot, settling time, perturbation peak, and recovery time.

5. The applicable scope given by Ref. [3] is obtained directly.

Acknowledgments

This project was supported by the National Natural Science Foundation of China (69804007) and the Science and Technology Phosphor Program of Shanghai (99QD14012).

References


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