Comparison of several well-known controllers used in process control

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Abstract
In this paper, several well-known design methods, PID control, Smith predictor, inferential control, internal model control, Dahlin controller, deadbeat control, and predictive control, are studied. A suboptimal Smith predictor is derived. The relationship among these methods is investigated. It is shown that these methods are equivalent to each other on certain premises. This explains why they are widely used in process control and provides insight into the work of bringing these techniques together to control system design. Examples are given to illustrate the result. © 2003 ISA—The Instrumentation, Systems, and Automation Society.

Keywords: Linear system; Time delay; PID controller; Smith predictor; Dahlin controller; Deadbeat control; Inferential control; Predictive control

1. Introduction

The control of processes involving time delay presents a continuing challenge to the control theorists. The nature of the time delay and the significant amounts of lag which can be introduced into the system response frequently make the use of conventional control algorithms a poor prospect. As early as 1953, Cohen and Coon [1] addressed the problem of controller design for system with time delay by correlating PID settings with model gain, time constant, and time delay. Low loop gain was required to avoid instability when time constant was small compared to the time delay, leading to poor system performance. Smith [2] suggested a time delay compensation scheme for single input/single output systems, now referred to as Smith predictor. Smith predictor is a simple and powerful control technique for processes with time delay. Its attractiveness comes from the fact that the design can be performed by using techniques applied to processes with rational transfer functions. However, it is sensitive to model mismatch and has poor disturbance rejection capability. How to overcome these shortcomings is the subject of numerous studies [3–6]. Other well-known techniques used in process control include continuous frequency domain, i.e., s domain design methods, such as inferential control [7] and internal model control (IMC) [8], and discrete domain methods such as the Dahlin controller [9], deadbeat control [10], and predictive control [11]. All of these methods have been applied to physical systems and achieved good response. Then, an interesting problem is what relationship exists among these methods? This problem was first presented by Zhang [12] and this paper will give a detailed discussion.

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This paper is divided into five sections. In Section 2, a suboptimal Smith predictor is derived based on the IMC method. Analytical formulas are given. In Section 3 other design techniques are analyzed and the relationship among these methods are investigated. The nominal performance and robustness are discussed in Section 4. Examples demonstrating the new design techniques are also presented. Conclusions are provided in Section 5.

2. Smith predictor design

A suboptimal design result of Smith predictor will be provided first. The structure of classical Smith predictor is shown in Fig. 1, where \( R(s) \) is the controller, \( G(s) \) is the actual plant, \( G_m(s) \) is the model, and \( G_{mo}(s) \) is the delay free part of \( G_m(s) \). The closed-loop transfer function between the setpoint \( r(s) \) and the output \( y(s) \) is

\[
T(s) = \frac{R(s)G(s)}{1 + R(s)[G_{mo}(s) - G_m(s) + G(s)]}.
\]

In the case of perfect modeling, i.e., \( G_m(s) = G(s) \), the closed-loop transfer function can be written as

\[
T(s) = \frac{R(s)G_m(s)}{1 + R(s)G_{mo}(s)}.
\]

This implies that the characteristic equation is free of the time delay and the primary controller \( R(s) \) can be designed with respect to \( G_{mo}(s) \). The achievable performance can thus be improved over a conventional system without delay compensation.

The Smith predictor can be related to IMC structure, which has been discussed by Morari and Zafriou [8] and Zhang [12]. Define the IMC controller

\[
Q(s) = \frac{R(s)}{1 + G_{mo}(s)R(s)}.
\]

Then the two schemes are equivalent to each other (Fig. 1). The closed-loop transfer function is

\[
T(s) = G_m(s)Q(s).
\]

The primary objective of any feedback control scheme is to make the difference between the controlled outputs \( y(s) \) and desired setpoint \( r(s) \) as small as possible. What is meant by “small” can be defined in terms of various performance specifications for the closed-loop system. In the IMC method, the performance objective is defined as \( H_2 \) optimal, i.e., \( \min \int [r(t) - y(t)]^2 dt \). Assume that the plant is stable. Factor the model

\[
G_{mo}(s) = G_{mo}^+(s)G_{mo}^-(s)
\]

such that \( G_{mo}^+(s) \) contains all right half plane zeros and is all pass. The suboptimal IMC controller can be expressed as

\[
Q(s) = J(s)/G_{mo}^-(s),
\]

where \( J(s) \) is a user-specified filter. Let \( n \) be the relative degree of \( G_{mo}(s) \). The filter with asymptotic tracking ability is usually selected as

\[
J(s) = 1/(\lambda s + 1)^n.
\]

Here \( \lambda \) is a positive real constant. The controller of Smith predictor can then be obtained through

\[
R(s) = \frac{Q(s)}{1 + G_{mo}Q(s)}.
\]

By tuning \( \lambda \), one can adjust the nominal performance and robust performance monotonically. In the case of perfect match, the nominal performance can arbitrarily approach optimality by decreasing \( \lambda \).

3. Other design techniques

3.1. Dahlin controller and deadbeat control

The Dahlin controller is a distinctive algorithm for the control of single input/single output (SISO) plant with time delay [9]. This design technique is influential in that it is simple to apply and can give
transfer function, from which the controller is
determined to be identical to a Dahlin controller.

Hence, after sampling, the suboptimal Smith predictor is

\[ C(s) = \frac{T(s)}{1 - T(s)G(s)}. \]  

(9)

Since the time delay is difficult to treat mathematically in continuous frequency domain, the design procedure is performed only in discrete domain. We will show that the Dahlin controller is only a special case of Smith predictor designed in continuous frequency domain. Assume that the plant is the first-order process with time delay. The suboptimal Smith predictor is

\[ R(s) = \frac{\tau s + 1}{K \lambda s}. \]  

(10)

The closed-loop transfer function can be written as

\[ T(s) = \frac{1}{\lambda s + 1} e^{-\theta_1 s}. \]  

(11)

Hence, after sampling, the suboptimal Smith predictor is identical to a Dahlin controller (Fig. 2).

The parameter \( \lambda \) in the Dahlin controller is consistent with the adjustable parameter in Smith predictor. When there is no model mismatch, \( \lambda \) is used to achieve desired nominal performance. If there exists model mismatch, increasing \( \lambda \) will enhance the robustness of the system. Since the relationship is monotonic, one can easily obtain the required response and parameter value by increasing \( \lambda \) from zero.

The Dahlin controller is initially designed to control the first-order plus time delay process. In a lot of literature, it is simply extended to control two-order or higher-order time delay processes with a first-order desired closed-loop transfer function. However, as discussed above, this kind of extension is not the best. The most desired closed-loop transfer function of a process with related degree \( i \) should be as follows:

\[ T(s) = \frac{1}{(\lambda s + 1)^i} e^{-\theta_1 s}. \]  

(12)

In this way, the Dahlin controller can be directly extended to the control of any minimum phase stable processes. If the process contains right half plane zeros, the desired closed-loop transfer function of a process with related degree \( i \) is

\[ T(s) = \frac{1}{(\lambda s + 1)^i} G_{mo}(s) e^{-\theta_1 s}. \]  

(13)

When \( \lambda \) tends to be zero, the Dahlin controller degrades to a deadbeat controller. This implies that deadbeat control can also be equivalent to the suboptimal Smith predictor.

3.2. Predictive control

Predictive control does not designate a specific control strategy but a very ample range of control algorithms developed in computer control systems. In fact, the digital form of Dahlin controller is just a predictive algorithm.

Suppose that the output of a minimum phase rational process is \( \tilde{y}(k) \) and the output of predictive model is \( y(k) \). Then the predicted output is

\[ y_p(k+i) = y(k+i) + \left[ \tilde{y}(k) - y(k) \right], \quad i = 1, 2, \ldots, P, \]  

(14)

where \( P \) is prediction horizon. The control objective is that the future output on the considered horizon should follow a determined reference trajectory, which is given by

\[ y_r(k+i) = \alpha \tilde{y}(k) + (1 - \alpha)y(k), \quad y_r(k) = y(k). \]  

(15)

Here \( \alpha = \exp( -T_s/\lambda) \), \( T_s \) is the sample time and \( \lambda \) is the time constant of the reference trajectory. Let \( L \) be the control horizon. The objective function is

\[ \sum_{i=1}^{P} \left[ y_p(k+i) - y_r(k+i) \right]^2. \]  

(16)
L control variables, \( u(k) \)'s, are then calculated by minimizing the objective function. If there exists time delay in the process, then the control sequence \( u(k) \) used for the process with time delay is just that calculated aimed at the delay-free process.

Compare the predictive model to the model, the reference trajectory to the closed-loop transfer function, and the objective function to the performance index, it can be seen that the results are very similar to that of the suboptimal Smith predictor. As a matter of fact, the model, the reference trajectory, and the objective function involved in not only predictive control, but also almost all feedback control methods.

Certainly, each predictive algorithm possesses its own form. Strictly speaking, not every predictive algorithm is equivalent to the Smith predictor. Consider an elemental predictive algorithm, model algorithm control (MAC). Assume that \( P=L=1 \). For the one step predictive control, solving objective function is very easy. Let \( y_r(k+1) = y_p(k + 1) \), then

\[
(1-\alpha)r = (1-\alpha)y_r(k) + y(k+1) - y(k).
\]

By Z transform, we have

\[
(1-\alpha)r(z) = (1-\alpha)y_r(z) + (z-1)y(z).
\]

On the other hand,

\[
y(z) = u(z)G(z), \quad y_r(z) = (1-\alpha)r(z)/(z-\alpha),
\]

therefore

\[
\frac{u(z)}{r(z)} = \frac{(1-\alpha)}{(z-\alpha)G(z)}.
\]

This implies that one step MAC is identical to the suboptimal Smith predictor. The reference trajectory is just the closed-loop transfer function and the objective function is just the \( H_2 \) performance index. They have identical manipulated variables. Similarly, it can be proved that dynamic matrix control (DMC) with \( P=L \) is also identical to the suboptimal Smith predictor for minimum phase processes.

Because of the intuition of continuous domain analysis, we can understand the model algorithm control profoundly from the following aspects: [5]

(a) Suppose that the order of the process is \( i \). To avoid the loss of information, the predictive model must have \( i \) predictive values.

(b) For an \( i \)-order process, the reference trajectory should have at least \( i \) values.

(c) To achieve the best control result, the process output should be close to the reference trajectory at every point, which implies that at least \( i \) control variables are needed.

3.3. Inferential control

Inferential control is proposed for the control of processes with unmeasured process outputs and unmeasured disturbances initially [7]. Consider the block diagram shown in Fig. 3. A typical process is given to the right of the dotted line, with one unmeasured process output \( y(s) \) and one secondary measured process output \( z \). The manipulated variable \( u(s) \) and the disturbance \( d_1(s) \) affect both outputs. The disturbance is considered to be unmeasured. The transfer functions in the block diagram indicate the relationships between the various inputs and outputs, and they are assumed to be perfectly known.

To the left of the dotted line is an inferential control system, where \( E(s) \) is the estimator, \( Q(s) \) is the controller, \( G_m(s) \) is the stable process model, \( M(s) \) and \( N(s) \) are the transfer function from the disturbance to the process outputs, and \( E(s) \) is the estimator. The job of \( E(s) \) is to combine its input and obtain an estimated effect of the disturbance \( d_1(s) \) on the process output. If the transfer function \( G_m(s) \) is equal to the transfer function \( G_j(s) \), then the signal entering the estimator is \( d_1(s)M(s) \). The controller manipulates the control effort to produce the opposite effect on the process output,

\[
u(s) = -d_1(s)M(s)E(s)Q(s).
\]

Cancellation is perfect if \( E(s) = N(s)/M(s) \) and \( Q(s) = G(s)^{-1} \), which is confirmed by the following input-output relation:
\[ y(s) = u(s)G(s) + d_1(s)N(s) \]
\[ = -d_1(s)M(s)E(s)Q(s)G(s) + d_1(s)N(s). \]  

(18)

Notice that the elements of the transfer function \( G(s) \) are lags whose numerator polynomials are lower than their denominator polynomials. This means that \( Q(s) \) contains elements that cannot be realizable. Thus a filter is added to solve the problem. Suppose that the process is given by an expression of the form
\[ G(s) = G_+(s)G_-(s)e^{-\theta s}, \]
where the all pass part \( G_+(s) \) contains all the right half plane zeros and \( G_-(s) \) the other. The filter is then defined as
\[ G_f(s) = \frac{G_+(s)e^{-\theta s}}{G_+(0)(\lambda s + 1)\pi}. \]  

(19)

Here \( n \) is selected such that the resulted controller is biproper. The idea of inferential control is afterwards applied to the case that the process output is measurable, i.e., \( M(s) = N(s) \), \( G(s) = G_+(s) \), and \( y(s) = z(s) \). Then Fig. 3 can be simplified to Fig. 1. The controller becomes \( Q(s) = G^{-1}(s)G_f(s) \). It is just the result of the suboptimal Smith predictor.

### 3.4. PID control

As discussed above, the Smith predictor structure can be equivalent to the classical unity feedback control loop. The controller of the unity feedback control loop is
\[ C(s) = \frac{Q(s)}{1 - G(s)Q(s)}. \]  

(20)

As \( \lambda \to 0 \), \( C(s) \) tends to be optimal. Because the controller \( C(s) \) involves the term \( e^{-\theta s} \) and any low-order rational controllers cannot approximate the time delay exactly, a PID controller can never reach the optimal control, or equivalently, exact analytical solution can never be obtained.

Suppose that there is a first-order process. The suboptimal Smith predictor gives the following unit feedback loop controller:
\[ C(s) = \frac{\tau s + 1}{K(\lambda s + 1) - e^{-\theta s}}. \]  

(21)

Let PID controller approximate the above controller. One way is to employ 1/1 order Padé approximation \([i.e., ~e^{-\theta s} = (1 - \theta s/2)/(1 + \theta s/2)]\) [16]. Then
\[ C(s) = \frac{1}{K} \frac{(\tau s + 1)(1 + \theta/2s)}{\lambda \frac{\theta}{2} s^2 + (\lambda + \theta)s}. \]  

(22)

In classical design methods, \( \theta \tau \) is a very important parameter and usually used for analyzing the closed-loop performance. It is seen that the effect of \( \theta \tau \) has been involved in the controller in the optimal design. For a second-order process, one can get the following suboptimal controller:
\[ C(s) = \frac{1}{K} \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{(\lambda s + 1)^2 - e^{-\theta \tau}}. \]  

(23)

To obtain a PID controller, the Taylor series or unsymmetrical Padé approximation can be used to approximate the time delay, then
\[ C(s) = \frac{1}{K} \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{\lambda^2 s^2 + (2\lambda + \theta)s}. \]  

(24)

There is little difference between the PID controller and the ideal controller in the low-frequency region. This implies that PID controller can also provide good performance for systems with time delay.

One can also let PID approximate the suboptimal Smith predictor or the suboptimal controller with time delay by numerical methods. However, the merit that the performance and robustness can be adjusted easily may be lost.

### 4. Discussions and examples

It is the dual purpose of control to change the dynamics of the given system for the better, and to be able to do this even if the system is not as well modeled as one would wish. The idea of confining the model-plant mismatch in the transfer function can be exploited according to more recent robustness criterion. Suppose the norm bounded uncertainty is defined by
\[ \frac{|G(j\omega)|}{|G_m(j\omega)|} - 1 \leq |L(j\omega)|, \forall \omega. \]  

(25)
Theorem 4.1. [8] The controller $C(s)$ provides robust stability if and only if

$$\|L(j\omega)T(j\omega)\|_\infty < 1, \forall \omega.$$  

A necessary and sufficient condition for robust performance is

$$\|W(j\omega)S(j\omega) + |L(j\omega)T(j\omega)|\|_\infty < 1, \forall \omega.$$  

Substituting $S(s)$ and $T(s)$ into the above equations yields

**robust stability:**

$$\|L(j\omega) \frac{1}{(\lambda j\omega + 1)^\alpha e^{-\theta s}}\|_\infty < 1, \forall \omega, \quad (26)$$

**robust performance:**

$$\|\frac{(\lambda j\omega + 1)^\alpha e^{-\theta s}}{s(\lambda j\omega + 1)^\alpha} + L(j\omega) \frac{1}{(\lambda j\omega + 1)^\alpha e^{-\theta s}}\|_\infty < 1, \forall \omega. \quad (27)$$

The results imply that arbitrary large stability margin can be obtained by the suboptimal Smith predictor. Theoretically, the parameter $\lambda$ should be chosen such that an optimal compromise between the performance and the robustness is reached. In the context of process control, $\lambda$ can be simply selected as $\alpha \theta$. The larger the parameter $\alpha$, the worse the nominal performance and the better the robustness. The range for $\alpha$ is from 0.1 to 1.2, which is a rule of thumb. A simple quantitative tuning procedure was developed by Zhang et al. [16]. It can be briefly stated as follows: increase the parameter $\alpha$ monotonically with the step 0.01 until the required nominal performance or robustness is obtained.

**Example 4.1.** Consider a blending process from Singh and Mcewan [13–15]. Two water streams enter a packed column at different temperatures, respectively. It is assumed that the two streams are mixed thoroughly in the column and the temperature of the blend is controlled by adjusting the hot water flow rate, the total flow being kept constant by automatic adjustment of the flow rate of cold water. The flow control loop is arranged to act at a much faster speed than the temperature control loop, so that interaction between the two loops is negligible. For changes small enough to allow the assumption of linear operation, the transfer function of the temperature control loop can be written as

$$G(s) = \frac{0.4e^{-24s}}{(13.8s + 1)(6.1s + 1)(3.9s + 1)}.$$  

The unit of time is second. As an example, it might as well take $\lambda = 6$ for suboptimal Smith predictor and $\lambda = 16$ for the equivalent PID controller. A step setpoint is introduced at $t = 0$ and a step disturbance at $t = 200$. The nominal system responses are shown in Fig. 4. It is seen that suboptimal Smith predictor provides distinct superior performance. In fact, if $\lambda$ is small enough, both the setpoint performance and disturbance rejection capability of the Smith predictor will tend to be optimal.

There always exists uncertainty in practice. When the uncertainty is larger than the error, similar responses will be obtained, since the difference of the two controllers comes from the approximation error. Assume that there are 20% uncertainty in process gain and 40% uncertainty in process time delay. The responses are shown in Fig. 5. This example shows that PID controller can also be used for controlling processes with large time delay and provide good response.

**Example 4.2.** Let us look at the diagram for a typical paper machine as shown in Fig. 6. It can be seen that the paper machine is divided into five sections: the head section, table and press section, dryer section, calender stack, and reel. Not shown in this figure is the stock preparation system where fibers are dispersed in water, the various other
materials in the papermaking suspension are added, and the suspension is delivered to the mixing tank. In the mixing tank and the headbox, the thick pulp is mixed with white water. Then the headbox delivers the diluted suspension of fibers to a fine mesh screen called the wire. The wire moves over the table where up to 95% of the water, which was in the headbox suspension, is removed by drainage through the wire. This produces a wet mat of fibers on the wire, which will become a finished sheet of paper after dried.

In the system, there are numerous control objectives such as basis weight, moisture content, stream pressure, consistency, etc., of which the most important is basis weight, i.e., the weight of one square meter of paper. By mechanistic analysis and identification, the dynamic model of the paper machine has been developed for basis weight control [17],

\[ G_m(s) = \frac{5.15 e^{-2.8s}}{1.8s + 1}. \]

According to the discussion of Section 2, the suboptimal internal model controller and inferential controller can be written as

\[ Q(s) = \frac{1.8s + 1}{5.15(\lambda s + 1)}. \]

The suboptimal Smith predictor is

\[ R(s) = \frac{1.8s + 1}{5.15s}. \]

All the three controllers are equivalent to the following unit feedback loop controller:

\[ C(s) = \frac{1}{5.15(\lambda s + 1)} - e^{-\lambda s}. \]

The \( C(s) \) is the result of the Dahlin controller in continuous frequency domain. These methods will result in the same manipulated variable and ISE (integral square error). If there is no uncertainty, the closed-loop response can tend to be optimal (the overshoot is zero and the rise time can be infinitely small).

**Example 4.3.** Consider the following process:

![Fig. 5. Responses of the uncertain systems (solid line: SP; dotted line: PID).](image)

![Fig. 6. Fourdrinier paper machine.](image)
\[ G_m(s) = \frac{e^{-0.8s}}{0.4s + 1}. \]

Let the sampling time be \( T_s = 0.4 \). We have
\[ G_m(z) = \frac{1 - e^{-0.4z}}{1 - e^{-0.4z}} \frac{e^{-0.8z}}{0.4z + 1} = 0.6321z^{-3} - 1. \]

The desired closed-loop response is
\[ T(s) = \frac{e^{-0.8s}}{\lambda s + 1} \]
of which the discrete domain form is
\[ T(z) = \frac{1 - e^{-0.4z}}{1 - e^{-0.4z}} \frac{1 - e^{-0.8z}}{1 - (1 - e^{-0.4z}) z^{-3}} \times 0.6321. \]

This is also the discrete domain solution for the suboptimal Smith predictor, internal model control, and inferential control. Its digital form is just the solution of the predictive control.

5. Conclusions

For a long time, the PID controller, Smith predictor, inferential control, internal model control, Dahlin controller, deadbeat control, and predictive control methods have been studied separately. This paper exploits relationships among them in a unified framework. Although some of these relationships have been considered by some literature, the interpretation of all of these relationships within this context is original. The main results of the paper are:

(a) The inferential control with the process out \( y \) is measurable and internal model control are equivalent to the suboptimal Smith predictor.

(b) The Dahlin controller, deadbeat control, and predictive control for minimum phase processes are equivalent to the suboptimal Smith predictor.

(c) The PID controller is approximately equivalent to the suboptimal Smith predictor, and the PID controller can also provide good response to a certain extent for system with time delay.

These results indicate that some of the methods developed in the early time are very effective, though their suboptimality has not been proved theoretically.

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References

[17] Zhang, W. D., Sun, Y. X., and Xu, X. M., Modeling of

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